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Novalis and Mathematics Revisited: Paradoxes of the Infinite in the Allgemeine Brouillon

Since Käte Hamburger's pioneering study *Novalis und die Mathematik* (1929, republished 1966), little work has been done on the relevance of Novalis's extensive comments about calculus to his philosophical conception of the infinite.[1] Recent contributions in the history of mathematics[2] as well as important advances in the understanding of Kant's philosophy of mathematics[3] make it possible to examine Novalis's theories about calculus in the context of the problems of mathematics and philosophy of his time. In my examination of the texts of Novalis, specifically the many notes from the *Allgemeine Brouillon* which refer to calculus, I will show that Hamburger's strictly neo-Kantian interpretation of No-

1 Novalis's mathematical work is treated briefly in the following works: Theodor Haering: *Novalis als Philosoph.* Stuttgart 1954; John Neubauer: *Symbolismus und symbolische Logik.* Munich 1978; Ulrich Gaier: *Krumme Regel: Novalis' „Konstruktionslehre des schaffenden Geistes" und ihre Tradition.* Tübingen 1970; Johannes Hegener: *Die Poetisierung der Wissenschaften bei Novalis.* Bonn 1975; and more extensively in Martin Dyck: *Novalis and Mathematics.* Chapel Hill 1960. However, none of these authors discusses in any detail Hamburger's claims or addresses specifically notions of infinity taken from Novalis's work with calculus. Erk Hansen (*Wissenschaftswahrnehmung und -umsetzung im Kontext der deutschen Frühromantik: Zeitgenössische Naturwissenschaft und Philosophie im Werk Friedrich von Hardenbergs (Novalis).* Frankfurt a.M. 1992.) comes closest to my thesis with his claim of a special priority of mathematics for the structure of the encyclopedia (pp. 393ff.). But his repeated claims that Novalis's analogous use of mathematics was independent of the mathematical research of his day (so that „die Mathematik hier also ausschließlich als Symbol benutzt wird, dessen Aussagekraft mit der Mathematik als Wissenschaftsdisziplin nichts zu tun hat," p. 396) ignores what I shall establish below: the question of the meaning of mathematics as a symbolic language was precisely the area of technical mathematical research which most interested Novalis.

2 Such as Ivor Grattan-Guinness: *The Development of the Foundations of Mathematical Analysis from Euler to Riemann.* Cambridge 1970; and Judith Grabiner: *The Origins of Cauchy's Rigorous Calculus.* Cambridge 1981.

3 Above all, the book by Michael Friedman: *Kant and the Exact Sciences.* Cambridge 1992.

valis's notes on calculus and their relevance for his "Weltanschauung" represents only one of a variety of perspectives which Novalis took from the greatest mathematical problems of his age.

In particular, I argue that Novalis's view of the infinite is more complex than a mere anticipation of the theories of the Marburg neo-Kantian School (Hermann Cohen, Paul Natorp, and Ernst Cassirer) and that this complexity is firmly based in the mathematical and philosophical debates of his time. Second, I examine how Novalis uses these debates as a source of analogy for investigations in other fields of inquiry and for the structure of knowledge as a whole in the context of his theory of the encyclopedia in the *Allgemeine Brouillon*. More precisely, in keeping with recent approaches to Novalis, I demonstrate that it is his attempts to synthesize the clashing perspectives on the meaning of calculus and to explore their complementarity, rather than to exploit the particular results of any one approach to calculus, which lead him to consider the mathematical theory of the infinite to be relevant in non-mathematical realms.

My basic disagreement with Hamburger can be summarized as a difference of interpretation of a single note from the *Allgemeine Brouillon*, namely: "Alle Vereinigung des *Heterogénen* führt auf∞" (III, 448). As I shall discuss in detail below, Hamburger takes the reference to the infinite in this note as proof of Novalis's use of the mathematical concept of the infinitely small and its power to comprehend difference by means of continuity, which is closely identified with the synthetic activity of the mind in the Kantian sense. I see the reference to the infinite, however, as oscillating in ambiguity between the infinitely small and the infinitely large, the latter representing the infinitely extended task of overcoming the irreducible discontinuity of the heterogeneous.

I begin with a critical discussion of Hamburger's article and its interpretation of Novalis. In a second section, I show what, in particular, interests Novalis in the mathematics of his day by rereading certain notes from the *Allgemeine Brouillon* in the context of the different theories of calculus at the end of the eighteenth century. In my final section, I demonstrate by means of a particular theme — the infinite series — the significance of this multiplicity of mathematical systems for the *Brouillon* project as a whole.

My treatment emphasizes the importance of understanding earlier theories of mathematics on their own terms and with regard to their own problems. Most of these problems have been "solved" using methods developed later. Though the retrospective approach can illuminate the underlying nature of the difficulties under consideration from the perspective of later theories, it should not necessarily be considered to be an authoritative one. In particular, I do not seek adumbrations of later developments in mathematics and physics (or even semiotics) in Novalis's writings.[4]

I. Hamburger's Interpretation of Novalis and Mathematics

To better understand Hamburger's argument, it is first necessary to mention the context in which she was writing in 1929. The principle task of her article was to defend Novalis against the accusations of Haym and Dilthey that his mathematical work represented the efforts of a dilettante and was thus not to be taken seriously. In addition, she was arguing against Heinrich Simon's interpretation of Novalis under the rubric of "magical idealism" (1906), attempting rather to incorporate him into the so-called "critical idealism" of the neo-Kantians. In part, this task was accomplished by strictly differentiating Novalis from his Romantic colleagues, such as Friedrich Schlegel, who are mentioned at several points in the article as taking a vague, unrigorous approach to the question of the infinite as opposed to the "methodical" mathematical approach taken by Novalis.[5] At certain points in her argument, it is also necessary for Hamburger to differentiate between two ways in which Novalis uses the concept of the infinite, one his mathematical, "methodical" use, the other his literary, metaphorical use.[6] Such a strategy leads Hamburger to attempt a reduction of certain statements to their "Sinn und Ursprung" and to

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[4] In this, I follow many others, including Peter Kapitza (*Die frühromantische Theorie der Mischung*. Munich 1968.) See also Michael Friedman who sees the usefulness of his investigation of Kant's theory of the sciences precisely in elucidating the logical problems he was trying to solve in the context of the science of his time, not in the context of our science (pp. xii, 56). However, there are also many examples of the opposite approach; compare, for example, Joyce Walker's article „Romantic Chaos: The Dynamic Paradigm in Novalis's *Heinrich von Ofterdingen* and Contemporary Science." In: *The German Quarterly* 66.1, 1993, 43-59. Thus, when I talk of contemporary science in this article, I refer to science contemporary to Novalis.

[5] For example, Schlegel is accused once of „schwärmen" in reference to the infinite. Käte Hamburger: „Novalis und die Mathematik." In: *Philosophie der Dichter*. Stuttgart 1966, 30.

[6] Hamburger, p. 57.

hermeneutic claims of the form "it is only possible to understand this note using this particular interpretation."[7]

For example, the first of the three sections of Hamburger's article attempts to reinterpret generally the significance of mathematics for Novalis. In particular, it attempts to strip the so-called "Hymnen auf die Mathematik"[8] of their religious flavor by establishing precise equivalents for terms which seem to suggest a mixture of contexts. For example, she claims that for Novalis the term "magic" was merely a technical abbreviation for Kantian "synthesis"; similarly, "Gott steht bei Novalis häufig für den Begriff des Unendlichgroßen."[9]

However, this method of reading tends to suppress the richness of Novalis's analogical thinking. In using a word taken from one realm of inquiry in the discussion of another, Novalis is not falling prey to a Romantic vagueness which arbitrarily ignores boundaries, but is rather pointing out underlying similarities and even, as we shall see below, creating a method for work between disciplines. Novalis himself identifies one of his tasks as the study of a new science: "Analogistik. Die Analogie — als Werkzeug, beschrieben und ihren mannichfaltigen Gebrauch gezeigt."[10] Such a task cannot be adequately understood by means of the reductions of meaning which Hamburger suggests. To examine the consequences of Hamburger's strategy, I will now turn to the second part of her article, in which she attempts to link Novalis's specific comments on calculus to the framework of the neo-Kantian theory of the infinitesimal.[11]

Hamburger proceeds by analyzing the use in Novalis of the three fundamental terms of neo-Kantian mathematical philosophy: function, infinitesimal, and continuity. It is difficult to isolate precisely the logical relationship Hamburger posits between these three concepts, because she tends to cite remarks by Cohen, Natorp, and Cassirer in the same breath, whereas their theories differ

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[7] Hamburger, pp. 18; 40, 50.

[8] III, 593; the name comes from Dilthey.

[9] Hamburger, p. 78.

[10] Novalis: *Schriften*. Ed. Paul Kluckhohn and Richard Samuel. Stuttgart 1977, III, 121.

[11] The final section of Hamburger's article is devoted to an analysis of the claim that Novalis, in his notes, anticipates later developments in mathematics and physics, in particular the theories of non-Euclidean geometry and relativity. As I have mentioned above, these claims do not fall within the scope of this paper. In addition, there can be no question of any actual influence.

precisely in this point.[12] In general, she claims that the function concept is only fully defined and brought to its complete development via the concept of continuity, which in turn depends on the notions of the infinitesimal calculus; this insistence on the unbreakable link between these three concepts forms a crucial part of her interpretation of Novalis and is also its greatest limitation. In order to present Hamburger's argument more fully, it is necessary here to explain briefly the basic notions of calculus along with their meaning in Hermann Cohen's theory of the infinitesimal (which has fallen into even more obscurity than other neo-Kantian theories).

The basic problem in calculus is that of describing in the form of a rule the relationship between two variable quantities whose assumption of a series of values can be correlated with each other. Thus, in describing the behavior of an object falling in the earth's gravitational field, one can measure for a variety of different time intervals how far the object has fallen in each, and then attempt to find a rule by which these two quantities (time and distance) can be correlated (the mathematical name for "rule" is, of course, function). It was axiomatic for the Marburg neo-Kantian school that this line of investigation constituted the fundamentally new method of modern physical science in contrast to Aristotelian models, and that Kant's philosophy, to a large extent, was the necessary response to and proper explanation of the realm of nature defined by such science.[13]

Calculus was invented at the end of the seventeenth century simultaneously in two different forms: Newton's method of fluxions and Leibniz's method of infinitesimals. In Newton's formulation, the more intuitive of the two, the basic concept of calculus is that of the rate of change of one quantity with respect to another, and its two basic procedures consist in finding this change, given the

[12] The lack of unanimity within the „Marburg school" is not discussed by Hamburger. In particular, Hermann Cohen criticized Cassirer's *Substanzbegriff und Funktionsbegriff* (Berlin 1910) for privileging the idea of the relation above that of the infinitesimal as the source of the „idealization of all materiality." See Dmitri Gawronsky: „Ernst Cassirer: His Life and His Work." In: *The Philosophy of Ernst Cassirer*, ed. Paul Schilpp. Evanston 1949, 21. The inadequacy of the neo-Kantian formulation for the standard modern conceptualization of calculus had been pointed out some years earlier by Bertrand Russell in his work *The Principles of Mathematics.*

[13] For the standard work on the differences between these two means of studying nature, see the aforementioned work by Ernst Cassirer.

law of correlation of the variables (differentiation), and given the rate of change, to find the law of correlation (integration). For example, given the law of gravitation (which indicates the acceleration), one can find the orbits of the planets and vice versa. Newton developed algorithms for these two procedures, and one of the major theoretical advances of calculus was to see that these two precisely inverse operations could be correlated with other pairs of mathematical operations. For instance, the operations of difference and summation in arithmetic[14] correspond to differentiation and integration, and, more impressively, so do the geometrical operations of finding the line tangent to a curve and finding the area under a curve. In other words, these algorithms could be used for a vast number of problems which were heretofore seen as unrelated.

Now what separates calculus, in the form in which it was developed, from other forms of mathematics is its use of the concepts of the infinitely small and the infinitely many (hence one of its names in the eighteenth century, the "analysis of the infinite"). Rates of change are not calculated over finite time intervals, but at a particular instant; the slope is defined not only for a line as a whole, but also at any particular point of a curve. Similarly, areas are found not only under straight lines (such areas can easily be broken up into simple rectangles and triangles), but also under curves and for curved surfaces (which seem to call for a division into infinitely many rectangles). Newton explains these problematical concepts by means of an appeal to our normal intuition of moving objects, but must also use such unclear concepts as, say, the speed of an arrow at a particular moment of its trajectory. In the approach of Leibniz, the notion of an infinitely small change is used to determine such quantities. For example, the speed of an object calculated at a particular moment of time is conceived as its speed as it moves an infinitesimal distance in a single instant. The question is then how to quantify such infinitely small periods of time. If the time is taken to be exactly zero, there is no motion at all; if we take only an extremely small amount of time, then the calculation is a mere approximation to one performed in a single instant. Thus, although both procedures, in practice, give rise to equiva-

[14] Stated simply: Start with a list of numbers; if you sum them progressively (i.e., the first, then the first two, the first three, etc.) and then take the difference of these sums, you will return to the original list; if you take the successive differences (the first minus 0, the second minus the first, the third minus the second, etc.) and add them together, this will also reconstitute the list.

lent and undeniably valuable results, neither Newton nor Leibniz is able to produce rigorous proofs that their algorithms yield the correct results. Such was the status of calculus even into the early nineteenth century: a set of procedures which could successfully solve problems, but only through the use of concepts which apparently could not be fully understood.

Hermann Cohen (1842-1918), some years later, returns to this crisis (as a philosopher) and reinterprets it in terms of the Kantian theory of consciousness, following certain of Kant's successors, most notably Salomon Maimon. Kant, who in his philosophy of mathematics seldom ventures beyond the theories of Euclid, is said by Cohen to speak of the modern conceptions of function and calculus in one single passage in the First Critique, namely, in the "Anticipations of Perception." Here, Kant claims that sensation, up to this point the last refuge of the unanalyzable, can also be quantified, in terms of its degree or intensive magnitude (Kant has in mind here the dynamic theory of matter, which defines the latter as an effect of repulsive and attractive forces which could vary continuously). Cohen interprets this claim to mean that reality — which he takes as a technical term for what we might call the scientifically constructed world — is created by the (scientific) mind by means of the method of the infinitesimal, the latter being its ideal element. Specifically, the degree of intensive reality in any particular sensation is equated with the infinitesimal, which is then integrated (read, synthesized) by the mind to produce the phenomena of mathematical physics. As an example, the mind produces the orbits of the planets as a function of time by showing how their motion is "generated" by the speed at each instant of the planets according to the law of gravitation (synthesis is needed, for the motion is never entirely present in one instant and hence does not exist as a unity outside of the synthesizing consciousness).[15] In other words, and this is Cohen's primary claim, the infinitesimal is one of the fundamental "categories" of scientific understanding through which (scientific) reality is produced. As he was fond of saying, "Die Sterne sind nicht am Himmel sondern in den Lehrbüchern der Astronomen."

To return to Hamburger, her claim is that mathematics was important for Novalis because he had discovered Cohen's theory of the infinitesimal and found in it a rigorous model by means of

[15] For this particular example, which is also used by Hamburger, see Paul Natorp: *Die logischen Grundlagen der exakten Wissenschaften*. Leipzig 1910, 208-213.

which he could conceptualize the Romantic belief in the creative nature of the mind's functioning. Mathematics, for Novalis, becomes the "Beweis und Abschluß" of transcendental idealism. Central for Novalis, on this view, is the function concept of neo-Kantianism, which Hamburger defines as serving the following purpose: "durch stetigen Übergang begriffsverschiedene Elemente unter einem Begriff zu vereinigen."[16] An example of this procedure (given by Hamburger and mentioned in Novalis: III, 422) is the mathematical understanding of the different conic sections (e.g., circle, ellipse, parabola) as being different cases of the same fundamental concept: by varying a parameter (the eccentricity) continuously, one can pass from a circle to an ellipse to a parabola (say, by starting with an ellipse in which the two foci coincide, i.e. a circle, and successively moving one focus to get larger and larger ellipses, which, in the limiting case of a focus at infinity, produces a parabola). This procedure stands in contrast to the Aristotelian notion whereby each geometrical shape is a separate concept abstracted from experience. Hamburger here follows Cohen (and not Cassirer) in insisting that continuity is only guaranteed by a reference to the infinite, in the form of the infinitesimal through which changes in a parameter are linked to and indeed produce the variety of geometric figures.

Hamburger then identifies these concepts taken from neo-Kantianism with the Romantic "Weltanschauung", particularly in its conception of the relationship between mind and nature and the correspondingly functional as opposed to substantial nature of its ontology. For example, the Schlegelian notion of *progressive Universalpoesie*, in its unification of all genres and sciences, is taken as exemplifying this concept of function. The generative property of the infinitesimal is thus identified with the creative power of the spirit in Romanticism. The flaw, however, in using Cohen's interpretation of calculus to describe the analogical procedure of Novalis lies precisely in Cohen's insistence on continuity and his corresponding reduction of the infinite to the infinitesimal. As I will show below in the third section, such an insistence ignores the crucial Romantic process of analogizing or, under another name, *Witz*. Thus, different fields of inquiry are connected by Novalis, not through a continuous progression of concepts, but rather through homonymity, the possibility of an infinite dispersal of meaning in a single term.

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[16] Hamburger, pp. 24, 26.

The weakest points in Hamburger's arguments are accordingly those moments when she tries to connect the discussions of the infinite in Novalis specifically to Cohen's theory of the infinitesimal. She attempts this by using notes from the *Allgemeine Brouillon* whose meaning she is forced to twist (sometimes by herself reading a term in Novalis as meaning the same thing as a homonymic term in Cohen), often by ignoring the contexts and resonances of the words involved (even to the point of only partially quoting the notes). Thus, when she tries to connect Novalis to the theory of the intensive magnitude as the differential of reality, she resorts to a note which seems to suggest the notion of degrees of reality in Novalis's use of terms such as "Graderhöhung". However, in the context of other notes,[17] it becomes clear that this term has to do with the degree of organization of a structure (in a sense deriving from Herder). Similarly the term "function" is often used by Novalis with biological reference (as in the function of an organ) rather than in its mathematical meaning.

As another example, consider Hamburger's reading of note 933,[18] where she elides a critical part of the note, namely, the last line printed here:

Die *Beweise* von Gott gelten vielleicht in *Masse* etwas – als Methode – Gott ist hier etwas, wie ∞ in der Mathematik – oder 0^{∞}. (*Nullgrade*) (Philosophie der 0.)
(*Gott* ist bald $1 \cdot \infty$ – bald $1/\infty$ – bald 0).

She sees this note, particularly the reference to the "Philosophie der 0", as confirmation that Novalis has understood Cohen's philosophy of the infinitesimal (which Cohen describes thus, "Auf dem Umweg des Nichts stellt das Urteil den Ursprung des Etwas dar"[19]). This it is, perhaps, in part (Novalis could have read the "Anticipations of Perception" and Maimon). But it is also at the same time an acceptance of other approaches to the infinite as well (hence the reference to taking the proofs of God not individually, but together). The many references in Novalis to the use of 0 in mathematics and philosophy presumably allude not (only) to Cohen, but (also) to the controversial theory of Leonhard Euler and his followers that Leibniz's infinitesimals were actually zeroes. Lazarus Bendavid went so far as to claim that not only in-

[17] Hamburger, pp. 39-40. She refers to note 554 (III, 362); my reference here is to note 633 (III, 381).
[18] III, 448; in Hamburger's article, p. 31.
[19] Hermann Cohen: *Logik der reinen Erkenntniss*. Berlin 1902, 69.

finitely small quantities were zero, considered arithmetically, but also infinitely large ones as well.[20] The meaning of the last line of the note is that calculus consists not of one particular, true method, but rather of a variety of approaches, and one must at times change between the perspective of the infinitely large and the infinitely small. The indeterminacy of the last line also argues against Hamburger's claim that "Gott" merely stands in for the infinitely large as does the explicitly analogical word "wie". We must therefore turn elsewhere in order to understand these aspects of the mathematical thought of Novalis, especially those which specifically relate to the project of the *Allgemeine Brouillon*, which he undertook during the period of his active study of mathematics.

II. The Basis of Novalis's Interest in the "Analysis of the Infinite"

Tremendous progress had been made in expanding the areas of application of calculus in the hundred years since Newton and Leibniz. It was a pardonable exaggeration for the author of a calculus textbook (in "Year VI," i.e., 1798) to write:

> De toutes les découvertes qui honorent l'esprit humain, l'analyse des infiniment petits, ou la méthode des fluxions, est peut-être la plus remarquable, soit par le caractère de l'invention, soit par la variété et l'importance de ses usages.[21]

Yet, the hesitation in the very definition of calculus, marked in the above quotation through the term "ou" (Newton or Leibniz), indicated that certain questions remained unresolved. As another indication of this same uncertainty, consider the prize question proposed in 1784 by the Berlin Academy. The text of the question reads, in part:

> ... de grands Analystes modernes avouent que les termes *grandeur infinite* sont contradictoire. L'Académie souhaite donc qu'on explique comment on a deduit tant de théoremes vrais d'une supposition *contradictoire*, et qu'on indique un principe sûr, clair, en un mot vraiment mathématique, propre a être substitué à *l'Infinie*.[22]

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[20] Lazarus Bendavid: *Versuch einer logischen Auseinandersetzung des mathematischen Unendlichen.* Berlin 1796, X.

[21] Charles Bossut: *Traités de calcul différentiel et de calcul intégral.* Paris 1798, i.

[22] Johann Schultz: *Versuch einer genauen Theorie des Unendlichen.* Königsberg 1788, 65-66.

In exploring why such questions would seem crucial after an entire century of successful mathematical practice, Grabiner points to the changing nature of post-secondary educational institutions and the corresponding need for textbooks as an impetus to treat these concerns within mathematics itself, rather than as solely philosophical questions.[23]

There is abundant evidence that Novalis came into contact with a variety of these textbooks (hence, theories) of calculus. In addition to private tutoring sessions with a fellow student during his time at the Freiberg Mining Academy (see the letter to his father, IV, 259), he had a substantial number of mathematics books in his possession.[24] These included both standard works in the field, such as the *Anfangsgründe der Analysis des Unendlichen* by the Göttingen mathematician and poet Abraham Gotthelf Kästner (1761), as well as contemporary attempts to fully re-systematize calculus, such as the *Theorie der analytischen Funktionen*, a text by the French mathematician Joseph Louis Lagrange (1797; German translation, 1798).

In order to determine more precisely the particular region of Novalis's interests, we can turn to the only surviving reading notes (III, 115-124) from mathematics textbooks (dating from the same time as the beginning of the *Allgemeine Brouillon*). These notes are based on Charles Bossut's *Traités de calcul différentiel et de calcul intégral* (1798) and Friedrich Murhard's more theoretical work *System der Elemente der allgemeinen Größenlehre* (1798). The excerpts from Bossut, after a brief quotation from the historical introduction, are taken exclusively from the introductory sections of the first four chapters, the sections entitled "notions générale," "principes généraux," etc., with no excerpts from the intervening text. This manner of selection shows that Novalis was far more interested in the basic definitions and principles of calculus than in its actual results (which, of course, are identical in all of the textbooks). This interest is confirmed by the excerpts from Murhard's book, which mainly concern disciplinary divisions within mathematics as well as general questions of method. Murhard's text consists in large part of plagiarized passages from other books, including works by Johann Schultz (mentioned once specifically by

[23] Judith Grabiner: „Lagrange and Analysis in the Eighteenth and Nineteenth Centuries." In: *Epistemological and Social Problems of the Sciences in the Early Nineteenth Century*, ed. Hans Jahnke and Michael Otte. Dordrecht 1981, 315.

[24] Listed in Dyck (note 1), p. 46; see also IV, 699.

Novalis), the friend of Kant and author of the work *Versuch einer genauen Theorie des Unendlichen* (1788). Novalis was thus interested in a variety of approaches to the problems of the foundations of calculus, i.e. methods of defining fundamental concepts and the systematic structure of mathematics as a whole, and also in attempts to link these approaches with the philosophy of Kant.

Indeed, another factor which led Novalis to this particular focus was the Kantian theory of mathematics itself. Kant and his followers define mathematics to be poetic (in the Greek sense of creative) rather than merely analytic[25] and are therefore much more interested in mathematical operations than mathematical objects, in constructions rather than abstractions. In this vein, Novalis writes:

> Allgemeiner Begriff der Multiplication – nicht blos der Mathematischen – so der Division, Addition, etc.
> Vorzüglich interessant ist diese philosophische Betrachtung der bisher blos mathematischen Begriffe und Operationen – bei den Potenzen, Wurzeln, Differentialen, Integralen, Reihen.[26]

To determine the reason for this special interest in mathematical foundations, it is necessary to consider the context of the *Allgemeine Brouillon*, which is a series of notes for a projected encyclopedia. Unlike the encyclopedias of the French Enlightenment, which were meant to be practical compendia of all of human knowledge (Novalis was well acquainted with D'Alembert's "Discours préliminaire"), Novalis saw his project, in agreement with Leibniz's idea of a *scientia generalis*, as an attempt to sketch the connections and analogies between different disciplines; the theory of such interdisciplinary connections he entitled "Encyclopaedistik." That he was interested specifically in disciplinary boundaries and their transgressions is evident from the fact that after several months of work on the *Allgemeine Brouillon*, he went back over his notes and classified them according to the "Wissenschaften" to which they belonged.

In one note (later classified as belonging to "Encyclopaedistik"), he writes, "Eine Wissenschaft gewinnt durch Fressen – durch Assimiliren andrer Wissenschaften etc. So die Mathematik z.B. durch den gefressenen Begriff des Unendlichen."[27] Besides itself being an example of an interdisciplinary statement – the

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[25] The *locus classicus* of this view is the description of Thales in the preface to the second edition of the *Kritik der reinen Vernunft* (B xi-xii).

[26] III, 260.

[27] III, 268.

term "Assimiliren" is an allusion to the theory of nutrition in contemporary biology which referred to the process by which an organism is able to produce more of its own matter out of heterogeneous material –, this note refers to the process by which a science takes a concept from another and assimilates it, i.e., makes it homologous to itself by situating the new concept within its own system of concepts. Below I will give an example from the Swiss mathematician Leonhard Euler which will illustrate this hypothesis. Here, what seems to be particularly important for Novalis is how the same term changes in meaning from one discipline or context to another. This consideration is the basis for the theory of *Analogistik* mentioned above.

Another question which this theory investigates is that of the conditions under which such exchange of concepts can occur; one might call this the historical part of the theory (as in many other fields, the end of the eighteenth century saw the first modern histories of mathematics, including one written by Bossut). In other words, the question is what makes a certain part of one science suggestive for another, either in calling for an interpretation from outside of the discipline itself or suggesting its use as a model for another discipline. The answer given by Novalis is that such exchange occurs most readily by means of a paradox; a paradox in one science, that is, a result which cannot be explained within the science itself or two mutually contradictory procedures which cannot be reconciled within the science, makes the science available for metaphorical appropriation. Novalis expresses this thought in the following note, "Die Verwandtschaft der Geometrie und Mechanik mit den höchsten Problemen des menschlichen Geistes überhaupt leuchtet aus dem atomistischen und Dynamischen Sektenstreit hervor."[28]

Hence, we see why Novalis's primary interest in calculus is in the question of foundations of calculus, the choice of basic concepts and the justification of basic procedures, rather than its extraordinary results. In his time, the latter were no longer considered to be paradoxical (for example, the question of "squaring" the circle had been solved by means of redefining the term "quadrieren" in terms of the procedure of integration). Novalis's view of interdisciplinarity led him to stress the unanswered questions in mathematics, which, according to his theory, were those relevant to other fields of inquiry.

[28] III, 387.

This perspective leads us to a somewhat different interpretation of the mathematical notes in the *Allgemeine Brouillon* from that given by Hamburger. I shall consider three different types of notes resulting from this point of view. First, Novalis frequently attempts to encapsulate the procedures of calculus in a single definition, almost always involving paradox. Thus, "Die Grundformel des Infinitesimal Calcüls $a/\infty \cdot \infty = a$"[29], which, in expressing the correlation and inverse nature of differentiation and integration, also contains the paradox of an infinite division of a quantity, which is then reconstituted through an infinite combination (an unmistakable reference to the question, going back to Zeno, of whether or not the continuum is made up out of points). In note 645, Novalis describes Leibniz's version of calculus as the measurement of the unmeasurable and the analysis of the indivisible. Hamburger downplays the paradoxical nature of such statements, saying of this note, "Hier ist es – ein wenig paradox – ausgedrückt,"[30] but the paradox is exactly the point. Finally, Novalis writes in note 981:

> Der Differentialcalcül scheint mir die *allgemeine Methode* das Unregelmäßige auf das Regelmäßige zu reduciren – es durch eine Funktion des Regelmäßigen auszudrücken – es mit dem Regelmäßigen zu verbinden ... – es mit demselben zu *logarythmisiren.*

Hamburger sees this note as an illustration of Cohen's claim that the infinitesimal serves to connect the generically different, but Novalis's claim goes beyond this: the infinitesimal joins the contradictory. Calculus reduces that which is not governed by a rule to that which is – and, thus, paradoxically is itself a rule for that which has none. Note that two of the metaphors (if they are metaphors!) used for this connection are themselves mathematical terms, "Funktion" and "logarythmisiren."

Connected to these notes about calculus are others in which Novalis discusses the two fundamental forms in which calculus was developed and discussed in the eighteenth century (as we have seen in the Bossut selection above). This topic is most evident in note 645:

> Die Verschiedenheit der Leibnitzischen und Neutonschen Vorstellungsart von der Rechnung des Unendlichen beruht auf demselben Grunde als die Verschiedenheit der atomistischen und Vibrations oder Aetherischen

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[29] III, 66. The following Novalis references in this paragraph are to III, 386 and III, 454.

[30] Hamburger, 32.

Theorie. Die Fluxion und das Differential sind die entgegengesezten Anschauungen des mathematischen Elements – beyde zusammen machen die mathematische Substanz aus.

Here, Novalis creates an analogy between the two original elaborations of calculus and the two competing Newtonian theories of the eighteenth century – certainly not the customary link seen between mathematics and physics.[31] Earlier, in note 634, Novalis mentions these mechanical theories and declares that "Beyde Systeme erklären sich gegenseitig"; following the conclusion of the later note, we might say that the most adequate explanation is the coincidence of opposing perspectives: one phenomenon requires two different and contradictory principles for its explanation (in note 634, this situation is compared to the "Geheimniß der Transsubstantiation"). Thus, in note 722: "(Doppelte), accidentelle mathematische Systeme – ihre Vereinigung im Infinitesimalcalcül. (Synthetische und Analytische Methode)." It is clear that this note again refers to Newton and Leibniz, first because it repeats the substance/accident distinction and second, because in note 646, Novalis identifies Newton's presentation with the synthetic method. The necessity of the conjunction of the synthetic and analytic methods refers to a tradition, ultimately deriving from Plato and reaching through Fichte, that true philosophical procedure consists of both analysis and synthesis. Here, then, the paradox lies not in one particular definition of calculus, but rather in the claim that two contradictory perspectives are necessary to fully illuminate its essence.

Such a link of perspectives is related to the emphasis given by Novalis in the *Allgemeine Brouillon* to the manner of presentation of theories (related to a more general theory of representation). For example, Novalis writes, "Der Vortrag der Mathematik muß selbst mathematisch seyn. / Mathematik der Mathematik."[32] In one sense, this note can be interpreted with regard to the self-referentiality of mathematics as a discipline as suggested by the last three words; consider the example above where the procedures of

[31] III, 386. Here, the differentials of Leibniz are identified with atoms, the fluxions of Newton with the vibrations of ether. Novalis may have been led to this comment by Kant's discussions in the *Metaphysische Anfangsgründe der Naturwissenschaft*, where he comes to the conclusion that both hypotheses, atomism and the ether, are logically consistent and hence the decision between the two cannot be made a priori; see Friedman, 218-19. The reference to Kant allows me to resist the formidable temptation to see here an anticipation of the wave-particle dualism of quantum mechanics.

[32] III, 245.

mathematics are described via metaphors taken from mathematics itself. Here, however, I will suggest another interpretation in keeping with the mathematical debates of the late 1790s, specifically related to Grabiner's claim that the manner of presentation of calculus conveyed at the same time the theory of its foundation. One criticism of Newton's fluxions which had been repeated throughout the eighteenth century was that they depended upon the concept of motion, which was foreign to mathematics.[33] The infinitesimals of Leibniz seemed also to be outside the domain of mathematics. Hence, the demand that mathematics be presented mathematically can be read as a demand to find secure foundations for calculus within mathematics itself. This is a perfect description of the book by Lagrange, whose principles were, according to the subtitle, "dégagés de toute considération d'infiniment petits, d'évanouissans, de limites et de fluxions, et réduits a l'analyse algébrique des quantités finies."

Such an approach to calculus is indicated by a series of notes in which Novalis defines pure mathematics as a "Bezeichnungslehre". A similar line of thought is evident in his repeated insistence that differentiation is akin to philosophical abstraction („Der Abstractions Calcül der Philosophie ist vollkommen dem Infinitesimalcalcül zu vergleichen" (III, 427); see also note 775).[34] Hamburger wishes to underplay this side of Novalis, because it places him on the wrong side of the division made by Cassirer, who links philosophical abstraction to an ontology of substances and Aristotelian logic as opposed to the modern scientific use of the function. But this side of Novalis is directly linked to Lagrange's theory of calculus as the formal manipulation of infinite power series. Lagrange attempted to avoid the "metaphysical" problems of such theories of calculus based on the notions of limits, infinitesimals, fluxions, etc., by reducing it to a series of purely formal rules of operations with symbols. Connected to this attempt was the Combinatorial School of German mathematicians (founded by Karl Friedrich Hindenburg, whose book Novalis owned and to whose methods there are frequent references in the *Allgemeine Brouillon*), whose members constructed a whole series of formulas, often arranged in tables, to describe the results of abstract operations such as the

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33 E.g., „Introduire le mouvement dans un calcul qui n'a que des quantités algébriques pour objet, c'est y introduire une idée étrangère." Joseph Louis de Lagrange: *Theorie des fonctions analytiques*. Paris 1881, 17.

34 III, 571; III, 427; III, 418.

multiplication of two infinite series.[35] In the work of Lagrange and these other mathematicians, variables were not seen as continuous and constructive, but rather in their symbolic, representative nature.

Thus, on the one hand, calculus is defined as a closed formal system of rules for the manipulation of expressions – the sort of system Novalis refers to as the goal of language in both the "Monolog" and the discussion of Sanskrit in *Die Lehrlinge zu Sais* – on the other hand, through the (post-)Kantian philosophy of calculus as *poiesis* discussed above, this creation of the mind has applicability a priori in that it constructs reality (as in the "Monolog," where the very closedness of the system of language magnifies its expressiveness). Here, the paradoxical implication is that by removing all reference to the outside of mathematics we obtain a theory which forms an isolated system, but in its very isolation is all the more strongly linked to reality as a whole. In the words of Novalis (here, paradox is replaced by "Wunder"):

Alles aus Nichts erschaffne Reale, wie z.B. die Zahlen und die abstracten Ausdrücke – hat eine wunderbare Verwandtschaft mit Dingen einer andern Welt – mit unendlichen Reihen sonderbarer Combinationen und Verhältnissen – gleichsam mit einer mathematischen und abstracten Welt an sich – mit einer poetischen mathematischen und abstracten Welt.[36]

Thus, common to all three types of notes, involving attempted definitions of calculus, the necessary complementarity of the approaches of Leibniz and Newton, and mathematics in its abstract, purely symbolic wholeness, is an emphasis on the paradoxes involved in the very foundations of calculus. This emphasis forms the basis of Novalis's interest in the subject.

III. A Case Study: Novalis and the Infinite Series

After having considered Novalis's discussion of what might be called the formal aspects of the state of calculus in the late 1790s,

[35] The connection between the work of Lagrange and the Combinatorial School is especially well developed in the article by Hans Jahnke: „A Structuralist View of Lagrange's Algebraic Analysis and the German Combinatorial School." In: *The Space of Mathematics: Philosophical, Epistemological, and Historical Explorations*, ed. Javier Echeverria et al. Berlin 1992. The work of the latter school was so limited in its merely formal abstraction and so confused by its introduction of non-intuitive symbols that it made no permanent mark in the later progress of mathematics.

[36] III, 440-41.

I shall now move to a specific matter of content within calculus, namely the question of the infinite series, as Novalis applies it to non-mathematical themes in the *Allgemeine Brouillon*. According to the model presented above, such an application presupposes that some paradox will be found within the mathematical discussion itself; hence, it is necessary first to discuss the problems associated with the infinite series.

Returning to the beginning of the eighteenth century, in 1703 the Italian monk and mathematician Guido Grandi (1671-1742) published a work on the geometrical squaring of the circle in which he exhibited the following argument[37]:

First, we start with the infinite series:

$$1 + 1/2 + 1/4 + 1/8 + \ldots$$

As can be easily justified[38], the sum of this series must be 2.

This result can be formally generalized (via an infinite long division) to yield the result:[39]

$$1/(1-x) = 1 + x + x^2 + \ldots,$$

which yields the previous equation, if we set x equal to 1/2. If we set x equal to -1, however, we arrive at the result:

$$1/2 = 1/(1-(-1)) = 1 + (-1) + (-1)^2 + \ldots = 1 - 1 + 1 - 1 + \ldots.$$

Grandi regrouped the terms on the right hand side to arrive at:

$$1/2 = 1 - 1 + 1 - 1 + \ldots = (1 - 1) + (1 - 1) + \ldots = 0 + 0 + \ldots = 0.$$

This leaves us with the paradoxical conclusion that 1/2 = 0 (which Grandi, in turn, used as a symbol for another paradox, the *creatio ex nihilo*).[40]

Most mathematicians were, of course, loathe to accept such a result, and many tried to resolve the seeming paradox (I will dis-

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[37] My discussion relies on Moritz Cantor: *Vorlesungen über Geschichte der Mathematik*. Vol. 3. Leipzig 1901, 365-6.

[38] Picture a number line. The first term represents starting at 1. Each additional term covers half of the remaining distance between your location and 2. Note that this is a mathematical version of Zeno's famous „dichotomy": in order for an arrow to move from point A to point B, it first must cover 1/2 the distance, then 1/2 of the remaining distance, and so on.

[39] This series also appears in notes taken by Novalis from a book by Abraham Gotthelf Kästner. See Hans-Joachim Mähl, „Zwei unveröffentlichte Handschriften aus der Berufstätigkeit Friedrich von Hardenbergs (Novalis)." In: *Jahrbuch des freien deutschen Hochstifts*, 1990, 131.

[40] Thus, we have an example of Novalis's claim above, that the paradoxical results of mathematics both demand and suggest analogy.

cuss below, in a different context, the response of Leibniz). The problem remained a source of dispute, when Euler (in his *Institutiones Calculi Differentialis* of 1755) suggested what he took to be a definitive solution: the infinite series should be regarded as being equal *by definition* to the expression from which it is derived (thus, the sum of Grandi's series is defined to be 1/2). In other words, Grandi's problem was merely one of terminology, which is easily solved, "wenn wir dem Worte Summe eine andere Bedeutung geben, als es gewöhnlich zu haben pflegt."[41] In one sense, this new definition is a retrospective justification of another of Euler's works, the famous *Introductio in Analysis Infinitorum* (1748), in which he derived remarkable formulas for infinite sums using a variety of formal manipulations of equally questionable expressions. In the terms I have discussed above, the re-definition represents a shift in interest from the particular results of mathematics (which can henceforth be used without fear of contradiction) to its foundational definitions and principles (which, in turn, will call for grounding). With its emphasis on the formal generation of a series through expansion of given terms, it is closely connected to the notion used by Novalis of the "Constructionsformel" of a series: "Wenige Bekannte Glieder, durch die man in Stand gesetzt wird eine unendliche Menge unbekannter Glieder zu finden – machen die Constructionsformel der Reihe aus."[42] The conception of a (known) part of a series generating the rule for the whole is crucial for Novalis; and the notion of an infinite series generated by a function was the basis for Lagrange's attempt to re-systematize calculus.

However, from a later point of view, Euler's definition represents an avoidance of the problem rather than a solution, since it merely shifts it onto the new terms "convergent" and "divergent," which remain to be suitably defined; as Euler writes: "Wenn also die Reihe eine convergierende Reihe ist, so stimmt dieser neue Begriff der Summe mit dem gewöhnlichen überein."[43] Within mathematics, the problem of the infinite series appeared to be solved with regard to calculations with particular series; the primary concern was the practical one of how fast a series converged in order

[41] Leonhard Euler: *Vollständinge Anleitung zur Differential-Rechnung*. Trans. Johann Michelsen. Wiesbaden 1981, p. 100. Dyck (note 1), p. 50, suggests that Novalis had access to this translation, originally published in Berlin in 1790.

[42] III, 68.

[43] Euler, p. 100.

to find a reasonable approximation to its limit, in order, say, feasibly to calculate the value of *pi* to a certain number of decimal places.[44] However, in the naive use of the term "convergent", one can still recognize a nagging question, going back to Bishop Berkeley's objections to the doctrine of fluxions (1734), namely whether an infinite series ever reaches its sum.[45] D'Alembert, for example, in an important discussion in the *Encyclopédie* (the article on the concept of limit), claimed that a variable approaches its limit without ever being able to reach it. Euler's definition circumvented this problem, though his use of the word convergent tacitly re-introduced it; for instance, it cannot be said that Grandi's series approaches its sum or its "sum" in the technical sense. Eventually this problem would be solved, by again proposing a definition which would make it irrelevant (as was done by the French mathematician Augustin-Louis Cauchy in the 1820s, according to whose definition Grandi's series has no sum at all), but at the time of Novalis, it was still an open question, especially for those interested in the "metaphysics" of mathematics.

In particular, the Kantian theory of mathematics had rather severe difficulties with infinite series and limit operations in general. As an example, consider the discussion of the square root of 2. For Kant, this quantity has no determinate ratio to 1 (i.e., it is an incommensurable) and thus cannot be fully given by a "number-concept" but only by means of a rule of approximation. To show that such a quantity actually exists, it is necessary that it be given to intuition in some form (in this case, in the construction of the diagonal of a square with side 1): "Only geometry can show the real possibility of the concept of $\sqrt{2}$."[46] Thus, an infinite series is of the form of a regulative concept, a rule for finding ever closer approximations to a quantity without ever being able to fully determine it. One might point here to a remark from Bossut's book which Novalis excerpted: "Unendliche Reihen, die man nicht summiren kann, oder die nicht summirbar sind, nennt man ... transcendente Größen."[47]

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44 For example, Euler derived a series for *pi* which enabled one to reach an approximate value far more quickly than by using Leibniz's famous series, which required a calculation of 10^{50} terms in order to reach one hundred decimal places; see Cantor, p. 668.

45 On this whole problem see Grabiner (note 2), p. 84-87.

46 Friedman, p. 118. For this section in general, I rely on Friedman, pp. 71-80, 111-112, 117-120.

47 III, 115.

Kant's approach to the larger question of limits appeals to intuition as well, in this case the pure intuition of motion. Found only in the "Anticipations of Perception" (explaining why Cohen also used this particular section of the text), Kant's discussion uses the vocabulary of Newton's theory of fluxions, in which convergence to a limit can only be conceptualized in terms of a point continuously approaching its limit: "That the limit of a convergent sequence exists is expressed by the idea that any such process of temporal generation has a terminal outcome."[48] Besides the lack of a consistent procedure to construct such limits (each of them must be visualized individually), Friedman points to the other major defect of this model, in that it cannot explain functions which are continuous but not differentiable, that is functions with sharp corners (such functions, and whether they occurred in nature, were already matters of dispute within mathematics at the time of Euler). The extreme cases of such functions are the self-similar fractal constructions of modern mathematics, which have, so to speak, sharp corners at every point; these functions are, in a way, infinitely concentrated generalizations of Grandi's series which changes direction at every term. The key point is that the Kantian notion of limit (or sum of a series) is based on a particular definition of continuity which excludes, precisely, changes of direction, or, in the language of Novalis, analogical discreteness. In particular, this Kantian-Newtonian approach can only explain the limit when it is actually reached in, say, a geometrical intuition (e.g., the infinitesimal generation of a curve), but cannot explain the algebraic operations with infinite quantities of modern calculus (such as the infinite generation of an algebraic series).

Käte Hamburger's treatment of the infinite series in Novalis remains within strictly Kantian parameters. In her view, the infinite series only attains its full meaning when it is combined with the notions of continuity and the infinitesimal and hence becomes the function: "das Reihenprinzip setzt sich vollends durch im Begriff der Funktion, die den Prozeßcharakter der Reihe erst zum prägnanten Ausdruck bringt."[49] It is in this discussion that Hamburger cites the first sentence of note 935 (here given in its entirety):

Alle Vereinigung des *Heterogénen* führt auf ∞. Theorie der Wahrscheinlichkeit – WahrscheinlichkeitsBeweise und Calcül – Quadratur des Unendlichen etc.

[48] Friedman, p. 74.
[49] Hamburger, p. 52, referring to III, 448.

Hamburger claims that the only possible interpretation of this note („erst unter diesem Gesichtspunkt erschließt sich auch die Bedeutung eines zunächst sinnlosen Satzes") relies directly on the neo-Kantian notion of continuity, which is the mind's ability in the unity of consciousness, "begriffsverschiedene Größen unter einem Begriff zu vereinigen, sie dadurch erst zu bestimmen."[50] However, she suppresses the ensuing connection to probability theory. Skipping over the semantic field of the overdetermined term "Heterogénen" (which is used by a variety of authors of the time, e.g. Hemsterhuis, Kant, and Schelling, as well as in texts on chemistry and galvanism), I would like to propose an alternate reading of the fragment based on a link drawn by Leibniz between probability and, strangely enough, Grandi's series.

Leibniz's response to Grandi's argument runs as follows: the partial sums of the series 1 – 1 + 1 – 1 +... alternate between 0 (if an even number of terms is taken) and 1 (for an odd number of terms). As there is no sufficient reason why the infinite sum should contain either an odd or an even number of terms, another principle must be employed here to arrive at a result, and that is the metaphysical principle of continuity. As nature is essentially continuous, when the series is expanded to infinity, the sum becomes the average between the zero and the one. In other words, in the infinite, the fundamental mathematical dualisms of Western metaphysics (the heterogeneous as such: from the odd/even of the Pythagoreans to the zero/one of the binary system) are reconciled. Leibniz gave a variety of explanations to justify the incursion of a metaphysical principle into a purely mathematical question, one of them being an analogy with probability:

> Wie die Wahrscheinlichkeitsrechnung vorschreibe, man habe das arithmetische Mittel, d.h. die Hälfte der Summe gleich leicht erreichbarer Größen in Rechunung zu ziehen, so beobachte hier die Natur der Dinge das gleiche Gesetz der Gerechtigkeit.[51]

Thus, the "∞" in Novalis's note seems not to refer to the infinitely small as the basis of the mind's ability to connect different phenomena, but rather to the infinitely large, the regulative idea of the never-ending progression, which in its very unendingness can transform the discrete into the continuous (as in Leibniz's theory of physical extension, where the infinite complexity of the mo-

[50] Hamburger, p. 50.
[51] Cantor, p. 367.

nads is perceived by us confusedly as continuous repetition of the same). That is, to invert Cohen's formula, the way to the infinitely small (continuity) is through the infinitely large. The connection to probability – which links expectation to the projected statistical average of an infinite number of trials – has remained unremarked by most of Novalis's commentators, including Dyck. There is no need to interpret this connection specifically with reference to Leibniz, since there is evidence (see notes 796, 798 of the *Allgemeine Brouillon*) that Novalis was familiar with the work of Condorcet on calculus and probability as well as the work of Laplace.[52]

Certainly the infinite referred to here is not reducible to that which dominates Hamburger's essay, invariably an infinitely small quantity related to continuity (e.g., "Das Unendliche des Bewußtseins, des Ich, wird nicht als eine quantitative, bis ins Unendliche ausgedehnte Größe aufgefaßt, sondern als eine intensive Erzeugungseinheit im Sinne des Differentials"[53]). Here, in contrast, we arrive at the infinitely small (continuity) by means of the infinitely complex process of overcoming heterogeneity. The means of this overcoming of the heterogeneous for Novalis is exactly the process of analogy. Indeed, several remarks of Novalis specifically link the discontinuity of nature with the discontinuity of thought, e.g. note 776 and note 183:

> Die Natur verändert sich sprungweise./ Folgerungen daraus. Synthetische Operationen sind Sprünge – (Einfälle – Entschlüsse.) Regelmäßigkeit des Genies – des Springers par Excellençe.[54]

More precisely, in other notes Novalis attempts to link the concept of the infinitely large and the infinitely small in a relation of "Wechsel," not of priority of one over the other. In note 290, Novalis writes:

> Die Unendlichkeiten verhalten sich wie die Endlichkeiten, mit denen sie *im Wechsel stehen*. Die Endlichkeit ist das *Integral* der Einen (Kleinen.) Unendlichkeit – und das Differential der andern (Großen) Unendlichkeit.

This theme is related to the notion of different orders of infinity and hence the pure relativity of the infinite and infinitesimal, com-

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52 III, 425f.; III, 69-71. The series 1 – 1 + 1 – ... is also used in a different context by Friedrich Schelling in the „Einleitung zu dem Entwurf eines Systems der Naturphilosophie" of 1799.

53 Hamburger, p. 41.

54 III, 273. The following references are to III, 291; III, 292.

mon in eighteenth century calculus (e.g., Kästner's description of a typical paradox of calculus: "Unendlich grosse Dinge die in Vergleichung mit andern Nichts, und unendlich kleine die in Vergleichung mit andern unendlich groß sind"[55]). The locus of transformation between these different infinities is the human, which forms, in terms reminiscent of Protagoras, "das Maaß aller Dinge ... das Organ ihres Contacts," the analogical center.

I will now present an example of Novalis's application of the "Wechsel" concept of the infinite in his analysis of the concept of infinite progression. As a starting point, I use note 447, which does not fit easily into Hamburger's interpretation and which she specifically labels as the exception which proves the rule.[56] For in this note, Hamburger claims, Novalis refers to the "method of exhaustion," i.e. the ancient equivalent of the limit approach, which avoids using the infinitesimal. However, in context, this note also allows for a different reading. It follows (and explicitly refers back to) a note involving the various notions of the term "solution" (chemical as well as logical). Novalis discusses a difficult problem and states that it is so difficult that its solution could only be envisioned "successive und Stückweise, d.h. in unendlichen Raum und in unendlicher Zeit." The key to progress is in its division. This leads to a reference to a passage in Kant's *Metaphysische Anfangsgründe der Naturwissenschaft* about chemical solution (Auflösung). In this passage, Kant discusses the possibility of a completed process of solution which would involve full inter-penetration of two chemical substances. Kant concludes that the idea of such a total solution is not self-contradictory, although it is beyond our understanding in the same way as the continuum is (cf. the second antinomy of the *Kritik der reinen Vernunft*):

Gegen die Möglichkeit dieser vollkommenen Auflösung und also der chemischen Durchdringung ist schwerlich etwas einzuwenden, obgleich sie eine vollendete Teilung ins Unendliche enthält, die in diesem Falle doch keinen Widerspruch in sich faßt, weil die Auflösing eine Zeit hindurch kontinuierlich, mithin gleichfalls durch eine unendliche Reihe Augenblicke mit Acceleration geschieht, überdem durch die Teilung die Summe der Oberflächen der noch zu teilenden Materien wachsen, und, da die auflösende Kraft kontinuierlich wirkt, die gänzliche Auflösung in einer anzugebenden Zeit vollendet werden kann. Die Unbegreiflichkeit einer sol-

[55] Abraham Gotthelf Kästner: *Anfangsgründe der Analysis des Unendlichen*. Göttingen 1761, ii.

[56] III, 329; Hamburger, p. 32.

chen chemischen Durchdringung zweier Materien ist auf Rechnung der Unbegreiflichkeit der Teilbarkeit eines jeden Continuum überhaupt ins Unendliche zu schreiben.[57]

Note the analogy to the Romantic concept of "Potenzierung" in this discussion; the result of each stage of dissolution is an increase in the rate of dissolution itself, which thus causes an exponential acceleration of the process. In the next note, Novalis talks about the application of this train of thought to the squaring of the circle; the problem can only be solved through an infinite approximation of the circle by polygons. The point is, in the symbolism of calculus, the inconceivable (because infinite) approximation is actually completed. Hence, we are suspended between two versions of the infinite series; in the first, it can only be completed through an infinite succession of steps, in the second, it is completed, though in an inconceivable, but mathematically operative manner. This process is analogous to Leibniz's description of the summing of the infinite series, which, by passing through the infinite, acquires a definite sum.

Thus, the meaning of calculus for Novalis, the paradox in which it lives, is precisely its ability to bridge the two models of unification: continuity based on the transition generated by the infinitely small and analogy which goes on to infinity. Calculus, in addition to representing "die Diskretion als Diskretion des Kontinuums",[58] is also "das Kontinuum als Kontinuum der Diskreten" (as mentioned, but not truly enacted by Hamburger) in the infinity of heterogeneity. Thus, in several notes, he speaks of the infinite as the ideal, the "Soll", the Fichtean aim of mathematics.[59] Yet, in the aptly numbered note 314 (the note talks of measuring the circle), he states (using the terms "Auflösung" and again, despite Hamburger's claim, "Approximationsprincipe") as regards the obstacles to the solution of these tasks:

Es liegt nur an der mangelhaften Natur, an den unvollkommnen Verhältnissen der gewählten *Constructionselemente* der Gegenstände dieser Aufgaben, (Elemente sind *Accidenzen*) daß sie nicht gelöst werden.

The term "Accidenzen" takes us back to note 722 where Novalis uses it to describe the two systems of calculus developed by New-

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[57] Immanuel Kant: *Metaphysische Anfangsgründe der Naturwissenschaft*. Vol. IV of *Kants Schriften*. Berlin 1902, pp. 530-31. For the relation of this problem to the chemical discourse of „Auflösung" at the time, see Kapitza (note 4), pp. 103-4.

[58] Hamburger, 58.

[59] III, 66; III, 296.

ton and Leibniz (as two attributes of mathematical "substance"); calculus itself, as we saw in the second section, is also subject to the same problem whereby the proper "accident" is lacking to fully ground it, yet its procedures function nonetheless.

In order to demonstrate this meaning of calculus in its full analogical splendor, let us now consider how Novalis uses the infinite series in the theory of the encyclopedia, i.e., of the unity of the sciences. For Hamburger, "gerade im Zusammenhang seiner Gedanken über Mathematik denkt er die Begriffe des Infinitesimalen, der Stetigkeit und der Funktion zu Ende bis zur Konzeption des Systems." However, again her argument for the "funktionale [...] Auffassung des Systems" is based on a partial reading; the note she refers to as the confirmation of the functional nature of the system[60] („Jede Wissenschaft kann durch reine Potenzirung in eine höhere, die philosophische, Reihe, als Glied und Function übergehen") complicates the reference to "Glied" and "Funktion" by also using them in the biological sense in addition to the purely mathematical. She also fails to cite the first part of the note where again it is analogy which works against continuity:

> Philosophie einer Wissenschaft entsteht durch Selbstcritik und Selbstsystem der Wissenschaft. (Eine Wissenschaft wird angewandt, wenn sie, als analoges Muster und Reitz einer specifischen Selbst(Nach)entwicklung einer andern Wissenschaft dient.)

Here, in addition, the idea of self-reference is related to the discrete, recursive nature of the infinite series, an infinite recursion which Hamburger acknowledges, but ultimately suppresses in favor of its dialectical opposite, the continuous generation of synthesis.

In turn, this self-referential recursive structure also governs the unification of sciences posited in the *Allgemeine Brouillon*. For, as Novalis writes in several passages, it is the task of any science to apply to, indeed to absorb all of the sciences, including itself:

> Doppelte Universalität jeder wahrhaften Wissenschaft – Eine entsteht, wenn ich alle andern Wissenschaften zur Ausbildung der Besondern benutze. – Die Andre, wenn ich sie zur Universalwissenschaft mache und *sie selbst unter sich* ordne – alle andre Wissenschaften, als ihre Modificationen betrachte.[61]

Novalis thus makes an analogy between the seemingly paradoxical phenomenon of the infinite series, that is an infinite pro-

[60] Hamburger, p. 62, 64, referring to III, 346.
[61] III, 269. The following reference is to III, 68.

gression which is generated by and, in a certain sense, contained in a part of itself („Wenige Bekannte Glieder, durch die man in Stand gesetzt wird eine unendliche Menge unbekannter Glieder zu finden“), and the claim that each science, as a sort of “Constructionsformel,” must incorporate every other science, therefore the entire universe, as well as explaining its own conditions of possibility. As an example of this process, consider the sequel of the note just quoted: “ReihenFormel einer ReihenFormelreihe.“

Thus, the unification of the sciences can only occur via an infinite series of discrete analogies, yet must at the same time be conceived of as a continuous system. It is this simultaneity of perspectives, this clash of viewpoints, which itself provides the analogy between *Encyclopaedistik* and the mathematics of the infinite series. As opposed to Descartes and Leibniz, for whom mathematics became a model for science because of its clarity and self-evidence, for Novalis it is precisely the mysterious ability of mathematics to function even in realms in which it must call upon a variety of seemingly incompatible methods to justify itself (because it involves a reference to the infinite) which makes it an appropriate model of knowledge as a whole.

As a final twist, let me note briefly that the mathematical version of this paradox (the part containing the whole) has itself been “resolved” by means of a new definition. The mathematical infinite (not as a quantity, but as the size of an aggregate) is now defined as that whose part is equal to the whole; in other words, the definition of infinity is precisely in the form of such traditional paradoxes as the question: are there more days or weeks in eternity. Yet, as if to preserve the open-endedness which I have discussed, the theory upon which this definition is based, set theory, has its own further paradoxes.

Thus, the infinite series and its paradoxes are a metaphor for the seemingly contradictory thought of an end to an infinite progress, of an infinitely large which stands “in Wechsel” with the infinitely small. The philosophical interchangeability of these two forms of the infinite has a long tradition (e.g., in Boethius, the identification of eternity and the moment); here, Novalis uses the various models of mathematics in his day to express it, in accordance with his theory of representation. Many other analogies to this double conception of the infinite can be found in Novalis’s writings, nowhere more paradoxically than in his balancing of the notions of the infinite perfectibility of humanity and of the golden age. Novalis speaks of the former on the occasion of his excerpt from

Bossut: “Jede Größe läßt sich, ohne Aufhören vermehren und vermindern,” to which he notes: “Indication der unermeßlichen Progressionsfähigkeit des Menschen – der Sinne, der Kräfte, etc.“[62] But, as we recall, calculus is the science which measures the immeasurable. Similarly, the infinite progression of the *Roman* is comprehended in the simultaneity of the *Märchen.*

[62] III, 118.